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Optimum Life Test Plans in Presence of Type-I Hybrid Censoring for Products Sold under General Rebate Warranty

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Abstract: The selection of an appropriate life test plan is extremely important for any product as it not just improves quality of the product but also reduces testing costs. In this approach however, the choice of suitable costs plays an important role. In this paper, a decision model is developed to determine optimal life testing plan by minimizing the relevant costs involved for non-repairable products sold under general rebate warranty. The life testing plan is developed in presence of Type-I hybrid censoring for products having Weibull distributed lifetimes. A constrained optimization approach is followed considering both producer's and consumer's risk and suitable analysis techniques are employed in obtaining the optimal solution. Monte Carlo simulation is conducted in order to illustrate that the specific risks (producer's and consumer's risk) are met. In order to study the sensitivity of the optimal solution due to mis-specification of parameter values and cost components, a well designed sensitivity analysis is incorporated using parameter estimates from real life Type-I hybrid censored data set.

Keywords: Life testing plans, Reliability, Acceptance sampling, Type-I hybrid censoring, General rebate warranty, Constrained optimization.

1 Introduction

In manufacturing and production industries, acceptance sampling is one of the widely employed techniques for industrial quality control. It helps manufacturing houses in tackling the problem of selection of a lot or batch of raw materials or any other component units. The disparity between the required and the real supplied manufactured goods quality can be diminished considerably with acceptance sampling plans Wu *et al* (2015). Quality of a product shapes beliefs of customers and hence can have an impact on sales which further justifies the importance of acceptance sampling plans. Typically an acceptance sampling plan can be described as follows. Let us suppose that a shipment of raw materials is received by a manufacturing unit. A sample is drawn from the lot of raw materials received and some pre-specified quality characteristics are examined. The information obtained from the inspected sample is wielded to reach the conclusion on acceptance or rejection of the lot.

The lifetime of a product is one of the essential quality features of consumer durable products. Reliability data are usually censored since the response values are not observable for all the units under study. So while testing the quality characteristic such as lifetime of a product we have to keep in mind that the quality attribute under study is not some instantaneously obtainable dimensional measurement. In life testing

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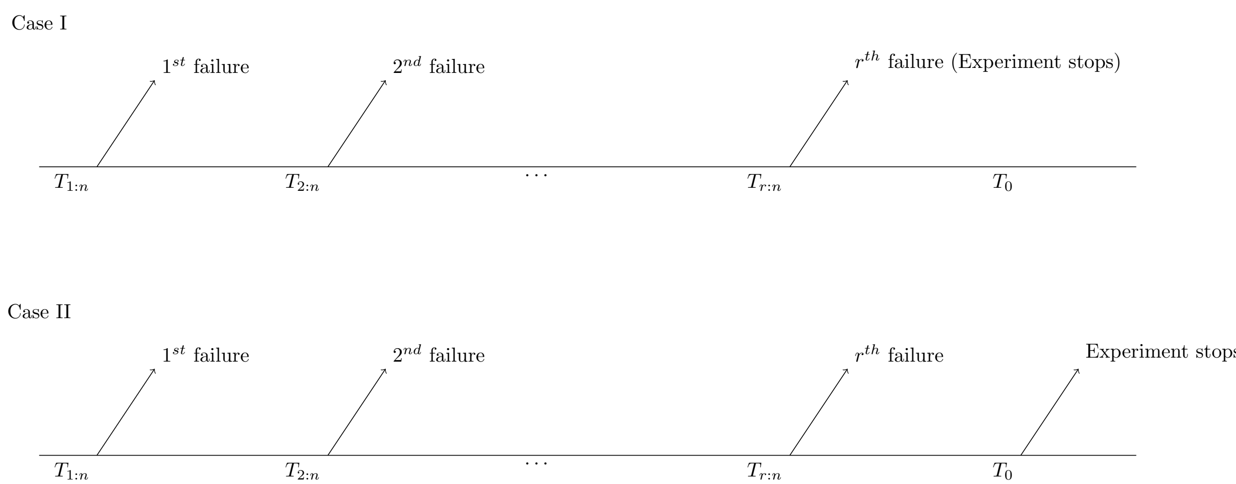
literature, the two most widely used censoring schemes are Type-I and Type-II censoring schemes. In Type-I censoring scheme, the test is aborted after a pre-decided time T; whereas in Type-II censoring, the termination of the test is subject to failure of a pre-fixed number of items r. The hybrid censoring scheme which is popularly known in the literature as Type-I hybrid censoring scheme was initially introduced by Epstein (1954) and can be considered as a mixture of Type-I and Type-II censoring schemes. It can be described briefly as follows: Let us consider n identical units are put on test. Now if $X_{1:n}, ..., X_{n:n}$ be the ordered lifetimes of the units put on test, then the experiment is aborted either when a pre-chosen number $r < n$ out of n items has failed or when a pre-determined time T has elapsed. Hence the life test can be terminated at a random time $T^* = min\{X_{r:n}, T_0\}$. One of the following two types of observations can be witnessed under Type-I hybrid censoring scheme.

Case I: $\{X_{1:n} < ... < X_{r:n}\}$ if $X_{r:n} < T_0$.
Case II: $\{X_{1:n} < ... < X_{d:n} < T_0\}$ if $d + 1 \leq r < n$ and $T_0 \leq X_{r:n}$.

Figure 1: Schematic illustration of Type-I hybrid censoring scheme.



Since life testing involve as well as influence various costs, it is rational for a decision maker to plan the test so as to minimize the average aggregate cost involved. Life testing plans under different censoring schemes have been studied extensively in the literature. Yeh (1994) used Bayesian approach to develop life test plans for products following exponentially distributed lifetimes under Type-I censoring setup. Balasooriya *et al* (2000) determines life test plans for products following Weibull distributed lifetimes under Type-II progressive censoring. The method followed by Balasooriya *et al* (2000) takes producer's and consumer's risks into account but no cost considerations are made in the design. A similar approach was followed by Bhattacharya *et al* (2015), where life test plans were developed for Weibull distributed products under hybrid censoring setup. In addition to that they also followed variance minimization approach to obtain optimum life test plans. Chen *et al* (2004a) proposed a general Bayesian framework for Weibull distributed product lifetimes with mixed censoring. Following a similar approach Chen *et al* (2004b) presented life test plans for products with exponentially distributed lifetimes using random censoring schemes. Bayesian approach with quadratic loss function was employed by Lin *et al* (2008) under hybrid censoring setup. Similarly other scholars have approached the problem of life testing through a combination of methods and censoring schemes, but while following a cost function approach very few papers in the available literature have included warranty cost which is an important cost for consumer durable products. Kwon (1996) was the first paper to include warranty cost as acceptance cost while designing optimal life testing plans for products with Weibull distributed lifetime under Type-II censoring setup employing a Bayesian approach. General rebate warranty policy was considered in the paper for calculation of warranty cost which is a combination of two most widely used elementary warranty policies for non-repairable products, free replacement warranty and pro-rata warranty. In free replacement warranty, a consumer can avail warranty services without any fee being incurred during the specified warranty period; whereas a pre-set proportion of the cost of repair is charged from a consumer on a pro-rata basis during the specified warranty period in case of pro-rata warranty. Following Kwon (1996); Huang *et al* (2008), Tsai *et al* (2008), and Hsieh and Lu (2013) included warranty cost in their studies under similar setup of Type-II censoring scheme. But to the best of our knowledge warranty cost has not been included for designing life testing plans in any other censoring setup.

In this paper we determine an optimum life test plan in presence of Type-I hybrid censoring using a cost function approach which has cost for products sold under general rebate warranty scheme having Weibull product lifetimes. A constrained optimization approach is inculcated to account for producer's and consumer's risk. The rest of the paper is organized as follows. In Section 2 we discuss in detail the framework of the acceptance sampling plan under Type-I hybrid censoring scheme for Weibull distributed product lifetimes. We describe the relevant costs involved and formulate the expected cost minimization problem in Section 3. In Section 4 we discuss the approach followed to obtain an optimum solution. In Section 5 we carry out the Monte Carlo simulation. It is interesting to note that appropriate variables can be used as a proxy of time to abort the experiment. One such example is used in Section 6 of this paper for the purpose of sensitivity analysis. The Lawless (2003) data pertaining to locomotive controls used in Section 6 is censored in terms of miles traversed instead of time. Finally, we put down our concluding remarks in Section 7.

2 Framework

We study life testing plan with Type-I hybrid censoring for products with Weibull distributed lifetime sold under general rebate warranty. Hence the lifetime X of a testing unit follows Weibull distribution with probability density function (pdf), $f_X(x)$ given by

$$f_X(x) = k\lambda^k x^{k-1} e^{-(\lambda k)^k}; x > 0, \tag{2.1}$$

where $k > 0$ and $\lambda > 0$ are the respective shape and scale parameters. The corresponding cumulative distribution function (CDF), $F_X(x)$ can be written as

$$F_X(x) = 1 - e^{-(\lambda x)^k}; x > 0. \tag{2.2}$$

If we consider the transformation $T = \ln X$, the corresponding CDF of the of the extreme value distribution of T is given by

$$F_T(t) = 1 - e^{-e^{\frac{t-\mu}{\sigma}}}; -\infty < t < \infty, \tag{2.3}$$

where $-\infty < \mu < \infty$ and $\sigma > 0$ are the respective location and scale parameters given by $\mu = -\ln\lambda$ and $\sigma = \frac{1}{k}$. Let $X_1, X_2, ..., X_n$ be the lifetimes of n units to be put on test which follow Weibull distribution given by (2.1). Hence $T_1, T_2, ..., T_n$ will be the corresponding log-lifetimes which follow extreme value distribution given by (2.3). Suppose the ordered lifetimes of these n units be given by $T_{1:n} \leq T_{2:n} \leq ... \leq T_{n:n}$. If we consider Type-I hybrid censoring framework, then the two random variables representing the number of failures and log-censoring time can be denoted by D and $\tau = min(T_{r:n}, T_0)$ respectively, where $T_0 = \ln X_0$ and X_0 is the censoring time. Accordingly the data can be represented by $(T_{1:n}, T_{2:n}, ..., T_{D:n}, D)$. The likelihood function can be written as

$$L(\mu,\sigma) \propto \prod_{i=1}^{d} f_T(t_{i:n})(1 - F_T(\tau_0))^{n-d}, \tag{2.4}$$

where $t_{i:n}$, d, and τ_0 are the observed values of $T_{i:n}$, D, and τ respectively. Using results from Park and Balakrishnan (2009), the Fisher information matrix can be written as

$$\ell(\theta) = \int_{-\infty}^{T_0} \left(\frac{\partial}{\partial\theta}\ln h_T(t)\right)^{'} \left(\frac{\partial}{\partial\theta}\ln h_T(t)\right) \sum_{i=1}^{r} f_{i:n}(t)dt; \tag{2.5}$$

where $h_T(t) = \frac{1}{\sigma}e^{\frac{t-\mu}{\sigma}}$ and $f_{i:n}(t) = i\binom{n}{i}\frac{1}{\sigma}e^{\frac{t-\mu}{\sigma}-(n-i+1)e^{\frac{t-\mu}{\sigma}}}\left(1-e^{-e^{\frac{t-\mu}{\sigma}}}\right)^{i-1}$ are the hazard and density function of T and $T_{i:n}$ respectively. The expression for $\ell(\theta)$ is of the form

$$\ell(\theta) = \begin{pmatrix} \ell_{11}(\theta) & \ell_{12}(\theta) \\ \ell_{21}(\theta) & \ell_{22}(\theta) \end{pmatrix};$$

where,

$$\ell_{11}(\theta) = \frac{1}{\sigma^2}\int_{-\infty}^{T_0}\sum_{i=1}^{r} f_{i:n}(t)dt,$$

$$\ell_{22}(\theta) = \int_{-\infty}^{T_0}\left(\frac{t-\mu}{\sigma^2} + \frac{1}{\sigma}\right)^2 \sum_{i=1}^{r} f_{i:n}(t)dt,$$

$$\ell_{12} = \ell_{21} = \frac{1}{\sigma}\int_{-\infty}^{T_0}\left(\frac{t-\mu}{\sigma^2} + \frac{1}{\sigma}\right) \sum_{i=1}^{r} f_{i:n}(t)dt.$$

Hence the variance-covariance matrix can be computed by inverting the Fischer information matrix as

$$\ell^{-1}(\theta) = \begin{pmatrix} \ell^{11}(\theta) & \ell^{12}(\theta) \\ \ell^{21}(\theta) & \ell^{22}(\theta) \end{pmatrix}.$$

While testing lifetime of a product as quality attribute, the lower specification limit is particularly important. Lower specification limit (LSL) is the lowest level of product quality that is within the acceptable range. Since in case of lifetime as a quality attribute, higher the lifetime of the product better is its quality. Hence we only need to be concerned with the LSL. Suppose the actual one-sided lower specification limit be L pertaining to items from (2.1), then the items with lifetimes less than L should be considered nonconforming and hence unacceptable. Since log-lifetimes are used while framing the model, therefore the fraction of nonconforming items, p, can be written as $p = Pr(T \leq L')$, where $L' = \ln L$. Using the lot acceptance criterion derived by Lieberman and Resnikoff (1955) we get the following expression

$$\hat{\mu} - k\hat{\sigma} > L'; \tag{2.6}$$

where $\hat{\mu}$ and $\hat{\sigma}$ are the maximum likelihood estimates of μ and σ respectively and k is acceptability constant. Typically r is fixed through the degree of censoring or proportion of censoring q, where $q = 1 - r/n$ (Schneider, 1989). The statistic $T = \hat{\mu} - k\hat{\sigma}$ is asymptotically normal with mean $E[T] = \mu - k\sigma$ and variance $Var[T] = \ell^{11}(\theta) + k^2\ell^{22}(\theta) - 2k\ell^{12}(\theta)$, where ℓ^{11}, ℓ^{22} and ℓ^{12} are elements of variance-covariance matrix and $\theta = (\mu, \sigma)$. So the standardized variate

$$U = \frac{\hat{\mu} - k\hat{\sigma} - (\mu - k\sigma)}{\sqrt{\ell^{11}(\theta) + k^2\ell^{22}(\theta) - 2k\ell^{12}(\theta)}} \tag{2.7}$$

is also asymptotically normal with mean 0 and variance 1. Therefore using arguments from Schneider (1989), the approximated OC curve can be represented by

$$\begin{aligned} \mathscr{L}(p) &= Pr(\hat{\mu} - k\hat{\sigma} > L'|p) \\ &= 1 - \Phi\left(\frac{\sigma(u_p + k)}{\sqrt{S}}\right); \end{aligned} \tag{2.8}$$

where, $S = \ell^{11}(\theta) + k^2\ell^{22}(\theta) - 2k\ell^{12}(\theta)$ and $u_p = \frac{L' - \mu}{\sigma}$ is the p^{th} quantile of the standard extreme value distribution corresponding to the nonconforming fraction $p = Pr((T - \mu)/\sigma \leq (L' - \mu)/\sigma)$ and $\mathscr{L}(p)$ is decreasing in p (Bhattacharya *et al*, 2015) and Φ is standard normal distribution function.

If we consider α and β as producer's risk and consumer's risk respectively, then by fixing points $(p_\alpha, 1-\alpha)$ and (p_β, β) on the OC curve we can obtain the value of k and also n for any known value of T_0.

The expression for k thus obtained can be written as

$$k = \frac{u_{p_\alpha} z_{1-\beta} - u_{p_\beta} z_\alpha}{z_\alpha - z_{1-\beta}},$$

and the value of n can be found out by solving the following expression for known value of T_0

$$\frac{S}{\sigma^2}\left(\frac{z_\alpha - z_{1-\beta}}{u_{p_\alpha} - u_{p_\beta}}\right)^2 = 1, \tag{2.9}$$

where z_α and $z_{1-\beta}$ are α^{th} and $(1-\beta)^{th}$ quantiles of standard normal distribution and u_{p_α} and u_{p_β} are ${p_\alpha}^{th}$ and ${p_\beta}^{th}$ quantiles of the standard extreme value distribution corresponding to the nonconforming fractions p_α and p_β respectively.

3 Determining the cost function

From the existing literature (Kwon, 1995; Hsieh and Lu, 2013) we can find an accordance that consistently emerges on the costs involved or impacted by life testing plan, following which we have involved in this study the following costs, 1. the cost of accepting a lot, 2. the cost of rejecting a lot, 3. the time-consumption cost, and 4. the inspection cost. Now if the products are sold under general rebate warranty policy, the decision to accept lot is going to effect the warranty cost. Hence warranty cost can be substituted for the cost of acceptance of a lot (Kwon, 1996; Huang *et al*, 2008; Tsai *et al*, 2008; Hsieh and Lu, 2013). The warranty cost thus adopted as acceptance cost is a combination of two warranty policies, free-replacement warranty and pro-rata warranty. The combination of the two warranty policies is best known in the literature as general rebate warranty. The mathematical expression for general rebate warranty is given by the following expression

$$c_w^*(x) = \begin{cases} c_w & x < w_1 \\ c_w \frac{w_2 - x}{w_2 - w_1} & w_1 \le x \le w_2 \\ 0 & x > w_2. \end{cases} \tag{3.1}$$

So, if the failure time is less than w_1, the cost incurred for free replacement is c_a. If the product has failure time between interval $[w_1, w_2]$, the cost incurred for pro-rata warranty is in proportion to the difference between failure time and w_2, which is decreasing in nature. If the failure time is beyond w_2, no warranty costs are incurred. Since we use log lifetimes, therefore according to general rebate warranty policy the cost of accepting an item with log-lifetime t is

$$c_a^*(t) = \begin{cases} c_a & t < \ln w_1 \\ c_a \frac{w_2 - e^t}{w_2 - w_1} & \ln w_1 \le t \le \ln w_2 \\ 0 & t > \ln w_2. \end{cases} \tag{3.2}$$

Therefore the expected warranty cost per unit is given by

$$w(\theta) = c_a \left(\frac{w_2 F_T(\ln w_2) - w_1 F_T(\ln w_1)}{w_2 - w_1} - \frac{1}{w_2 - w_1} \int_{\ln w_1}^{\ln w_2} e^t f_T(t) dt \right). \tag{3.3}$$

Hence the expected warranty (acceptance) cost if n out of N items are put on test is given by

$$C_w = (N - n) w(\theta) \left(1 - \Phi\Big(\frac{\sigma(u_p + k)}{\sqrt{S}} \Big) \right). \tag{3.4}$$

From the literature, rejection cost usually is taken as cost due to units that are not tested (Hsieh and Lu, 2013). Thus if c_r is the cost per unit for the items that are not put on test, then the average cost of rejecting a lot is given by

$$C_r = (N - n) c_r \Phi\Big(\frac{\sigma(u_p + k)}{\sqrt{S}} \Big). \tag{3.5}$$

Using results of average time taken during test from Bhattacharya *et al* (2014), the expected log-time of the test can be written as

$$\begin{aligned} E[\tau] &= E[min(T_{r:n}, T_0)] \\ &= T_0 P(T_{r:n} \ge T_0) + E[T_{r:n} | T_{r:n} < T_0] P(T_{r:n} < T_0) \\ &= T_0 \Big(1 - \sum_{j=r}^{n} \binom{n}{j} F_T(T_0)^j \big(1 - F_T(T_0)\big)^{n-j} \Big) + r \binom{n}{r} \int_0^{T_0} t F_T(t)^{r-1} \big(1 - F_T(t)\big)^{n-r} f_T(t) dt. \end{aligned} \tag{3.6}$$

Thus if c_t be the cost per unit, the expression for expected time consumption cost is given by $C_t = c_t E[\tau]$. Also if c_i is the unit cost of inspection, the average cost of inspection can be written as $C_i = n c_i$. The aggregate cost function is

$$\begin{aligned} TC(n, T_0) &= C_w + C_r + C_t + C_i \\ &= (N - n) w(\theta) \left(1 - \Phi\Big(\frac{\sigma(u_p + k)}{\sqrt{S}} \Big) \right) + (N - n) c_r \Phi\Big(\frac{\sigma(u_p + k)}{\sqrt{S}} \Big) + c_t E[\tau] + n c_i \\ &= (N - n) \left(w(\theta) + \big(c_r - w(\theta)\big) \Phi\Big(\frac{\sigma(u_p + k)}{\sqrt{S}} \Big) \right) + c_t E[\tau] + n c_i \end{aligned}$$

Therefore, the optimal design problem can be expressed as follows:

$$\text{minimize } TC(n, T_0)$$

$$\text{subject to } \frac{S}{\sigma^2}\left(\frac{z_\alpha - z_{1-\beta}}{u_{p_\alpha} - u_{p_\beta}}\right)^2 - 1 = 0.$$

The equality constraint as also shown in (2.9) ensures that the already agreed upon values pertaining to producer's and consumer's risks are being maintained.

4 Determining the optimal solution

The optimization problem for determining the optimal life testing plan stated in the aforementioned section is fairly complex in nature. It is important to note that the complexity is enhanced by its twofold character of being a nonlinear mixed integer programming problem. Due to the complex nature of the mathematical functions involved in framing various costs that constitute the objective function, integer inputs for the values of n are required. Hence to simplify the problem, instead of using n as a decision variable we use $p_n = n/N$. To retain the integer nature of n, we replace n with $\lfloor p_n^* N \rfloor$, where $\lfloor . \rfloor$ represents greatest integer or the floor function. The continuous nature of p_n ($p_n \in [0, 1]$) transforms the problem to a nonlinear programming problem where the traditional algorithms such as augmented Lagrangian can be used to find the optimal solution. Therefore the transformed problem can be written as follows

$$\text{minimize } TC(p_n, T_0)$$

$$\text{subject to } \frac{S}{\sigma^2}\left(\frac{z_\alpha - z_{1-\beta}}{u_{p_\alpha} - u_{p_\beta}}\right)^2 - 1 = 0.$$

In order to extract the optimal value of n from the solution obtained by solving the above problem we again take the help of the floor function. The procedure followed to solve the problem can be summarized using the following steps:

Step 1: Take $p_n = \frac{n}{N}$ as a decision variable.
Step 2: Replace n with $\lfloor p_n^* N \rfloor$ in the objective and the constraint function.
Step 3: Minimize the objective function with respect to the given constraint to find the optimal values of (p_n^*, T_0^*).
Step 4: Obtain $n^* = \lfloor p_n^* N \rfloor$ and $X_0 = e^{T_0}$ to find the optimal design (n^*, X_0^*).

The values of p_α and p_β are usually determined through joint accordance of the producer and the consumer. But for the purpose of our study we used the values from MIL-STD-105D (U D of Defense, 1963) which is a common practice followed in the literature (Schneider, 1989; Balasooriya and Low, 2004; Bhattacharya *et al*, 2015). For computational purpose we have used the values of the parameters as $\mu = 0$ and $\sigma = 1$. We then found the expression for total cost considering the following unit costs $c_a = 0.15$ (unit cost of acceptance), $c_r = 0.80$ (unit cost of rejection), $c_t = 0.08$ (unit cost of time consumption) and $c_i = 0.05$ (unit cost of inspection). The *auglag* function from *nloptr* package in *R* 3.2.2 was used to solve the problem. The solutions thus obtained are stated in Table 1.

It can be observed from the results that the optimum sample size (n^*) increases with increase in the degree of censoring (q) whereas the optimum censoring time (X_0^*) decreases. Figure 2 portrays the aforementioned observations graphically. The decrease in optimum total cost with increase in degree of censoring can be attributed to the observed decrease in censoring time as well as decrease in proportion of failures which follows from the relationship $q = 1 - r/n$.

In the above illustration the lot size (N) is assumed to be 500. We observe the effect in optimal design with change in lot size from the following table (Table 2) keeping the degree of censoring fixed at 0.5. We can observe that with change in lot size the optimal design does not show any major difference.

5 Monte Carlo simulation

A rigorous simulation study is conducted to validate if the stated risks are being met. The study is conducted for plans computed in Tabe 1. Keeping $n, k, X_0, \alpha, \beta, p_\alpha$ and p_β fixed, 10,000 data sets are simulated. The maximum likelihood estimates are obtained using equation (2.4) for each of the data sets. Now if we consider $L^{'} = {F_T}^{-1}(p_\alpha)$ and use the lot acceptance criterion $\widehat{\mu} - k\widehat{\sigma} > L^{'}$ to reject the lots, the proportion of rejection should come close to α. If we repeat the same considering $L^{'} = {F_T}^{-1}(p_\beta)$, the proportion of acceptance should come close to β. Table 3 shows the results of the simulation study. It can be seen that the estimated values ($\widehat{\alpha}$ and $\widehat{\beta}$) are closer to the stated risks.

6 Sensitivity analysis

In the process of development of the aforementioned sampling plan we rely on the parameters of the extreme value distribution of T. So the optimum design depends on the choice of the parameters of the distribution. Hence to investigate the effect of mis-specification of parameters in the optimum design and the total cost, we incorporate a sensitivity analysis study. For the purpose of this study we have used a historical real life data set from Lawless (2003). Each point of the data set represents the number of thousand miles at which different locomotive controls have failed in a life testing experiment. The data set involves a sample size (n) of 96 and the test was aborted either when 135 thousand miles (T_0) elapsed or when 37 failures (r) were observed. So as we can observe that number of traversed miles is taken as a proxy for failure times for the life testing experiment. The data depicting the failure times (in thousand of miles) of the units failed during the life test experiment is represented as follows:

22.5, 37.5, 46.0, 48.5, 51.5, 53.0, 54.5, 57.5, 66.5, 68.0, 69.5, 76.5, 77.0, 78.5, 80.0, 81.5, 82.0, 83.0, 84.0, 91.5, 93.5, 102.5, 107.0, 108.5, 112.5, 113.5, 116.0, 117.0, 118.5, 119.0, 120.0, 122.5, 123.0, 127.5, 131.0, 132.5 and 134.0.

For the purpose of our analysis we estimate the parameters of the distribution given by equation (2.3) with respect to the aforementioned data set. Note that in addition to the above data points the data set consists of 59 more data points which assumes a value of 135 (censoring time). The maximum likelihood estimation is done using the expression for likelihood function obtained from (2.4). We also obtain the standard errors of the estimates and hence using estimates $\pm$ standard error we find three set of values for each parameter. Since we have two parameters involved in the process with three values attributed to each, therefore using different combinations we arrive at nine pairs of parameter values. Now using the $\alpha = 0.05$, $\beta = 0.1$ and three pairs of (p_α, p_β) values from MIL-STD-105D (U D of Defense, 1963), as has been used earlier, we find the optimal design corresponding to each of the nine set of parameters. Note that the degree of censoring in this case is considered to be $q = 0.614583$ using the fraction of failed items in the data set. Table 4 gives us the results thus obtained through the process. From Table 4 we can observe that the optimal design does not show any great deviation or trend with slight change in parameter values. But on the other hand, a clear trend emerges from the values of optimum cost. It can be seen that with the increase in parameter μ (keeping σ fixed) the optimum total cost decreases but with the increase in parameter σ (keeping μ fixed) the optimum total cost increases. Now the change in total cost may perceptively appear insignificant but it should be kept in mind that all the unit costs are considered within an interval $[0, 1]$ and hence any change in first or second decimal values of optimum total cost cannot be considered negligible and hence insignificant.

7 Conclusion

In this work, a method has been proposed to arrive at optimum reliability acceptance sampling plans under Type-I hybrid censoring. We have considered Weibull lifetime models in the context of our study, however under the ambit of developed methodology other lifetime distributions of log-location scale family can also be used. The work tries to formulate optimum reliability acceptance sampling plans from a management perspective which makes it valuable in dealing with real life problems pertaining to product quality management. As a scope for future research, the proposed method can also be studied under other censoring schemes. Many a times it may be realistic to assume that the parameters involved arise out of some prior distribution because of uncertainty engaged in the parameter values. Hence for future research the problem can also be studied under a Bayesian approach.

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Appendix A

Table 1: Type-I hybrid censored reliability acceptance sampling plans for given values of $q, \alpha, \beta, p_\alpha$, and p_β.

(α, β)			(0.05, 0.1)				
(p_α, p_β)	q	k	p_n^*	n^*	T_0^*	X_0^*	TC^*
	0.1	3.129706	0.1005	50	4.2131	67.5670	81.7436
	0.2	3.129706	0.1081	54	3.0617	21.3630	81.4389
(0.02090, 0.07420)	0.3	3.129706	0.1196	59	2.0909	8.0924	80.5850
	0.4	3.129706	0.1293	64	1.7169	5.5670	79.8420
	0.5	3.129706	0.1362	68	1.3538	3.8719	79.5403
	0.1	4.509205	0.1709	85	4.7665	117.5056	84.6080
	0.2	4.509205	0.1875	93	3.8835	48.5947	83.4286
(0.0190, 0.05350)	0.3	4.509205	0.2043	102	2.8297	16.9419	81.9584
	0.4	4.509205	0.2214	110	1.5950	4.9283	80.8920
	0.5	4.509205	0.2404	120	1.1415	3.1314	79.4685
	0.1	2.801873	0.1136	56	4.6910	108.9661	70.0339
	0.2	2.801873	0.1229	61	3.4177	30.5019	69.4926
(0.03190, 0.09420)	0.3	2.801873	0.1308	65	2.8921	18.0324	69.0587
	0.4	2.801873	0.1382	69	2.6834	14.6361	68.6409
	0.5	2.801873	0.1476	73	2.1496	8.5817	68.2282

Table 2: Type-I hybrid censored reliability acceptance sampling plans for different lot sizes (N) with $q = 0.5$.

(p_α, p_β)	N	p_n^*	n^*	T_0^*	X_0^*	TC^*
(0.02090, 0.07420)	300	0.2285	68	1.3186	3.7384	44.2756
	500	0.1362	68	1.3537	3.8719	79.5403
	1000	0.0682	68	1.2912	3.6373	167.7019
	1500	0.0457	68	1.0592	2.8841	255.8635
(0.0190, 0.05350)	300	0.4021	120	1.2927	3.6426	40.7520
	500	0.2404	120	1.1415	3.1314	79.4685
	1000	0.1208	120	1.3746	3.9535	176.0174
	1500	0.0801	120	1.1147	3.0488	272.6502
(0.03190, 0.09420)	300	0.2444	73	2.0734	7.9515	37.9654
	500	0.1476	73	2.1496	8.5817	68.2282
	1000	0.0744	74	2.3835	10.8427	143.7471
	1500	0.0489	73	2.2784	9.7607	219.5422

Appendix B

Table 3: Simulation results obtained using Type-I hybrid censored reliability acceptance sampling plans for $\alpha = 0.05$ and $\beta = 0.1$.

(p_α, p_β)	q	n^*	X_0^*	$\widehat{\alpha}$	$\widehat{\beta}$
	0.1	50	67.5670	0.041	0.124
	0.2	54	21.3630	0.043	0.121
(0.02090, 0.07420)	0.3	59	8.0924	0.043	0.118
	0.4	64	5.5670	0.045	0.120
	0.5	68	3.8719	0.044	0.117
	0.1	85	117.5056	0.043	0.118
	0.2	93	48.5947	0.044	0.118
(0.0190, 0.05350)	0.3	102	16.9419	0.044	0.119
	0.4	110	4.9283	0.042	0.117
	0.5	120	3.1314	0.045	0.120
	0.1	56	108.9661	0.046	0.121
	0.2	61	30.5019	0.046	0.123
(0.03190, 0.09420)	0.3	65	18.0324	0.047	0.121
	0.4	69	14.6361	0.046	0.120
	0.5	73	8.5817	0.047	0.119

Appendix C

Table 4: Results of sensitivity analysis using Lawless (2003) failure data of locomotive controls where $q = 0.614583$.

(p_α, p_β)	$\widehat{\mu}, \widehat{\sigma}$	${p_n}^*$	n^*	T_0^*	X_0^*	
	(5.193145151, 0.418876401)	0.1770	88	4.1657	64.4389	25.2745
	(5.2010132, 0.418876401)	0.1776	88	4.1735	64.9480	25.1509
	(5.208881249, 0.418876401)	0.1773	88	4.1814	65.4610	25.0294
	(5.193145151, 0.4231725)	0.1777	88	4.1552	63.7635	25.4073
(0.02090, 0.07420)	(5.2010132, 0.4231725)	0.1774	88	4.1630	64.2672	25.2828
	(5.208881249, 0.4231725)	0.1774	88	4.1709	64.7748	25.1604
	(5.193145151, 0.427468599)	0.1754	87	4.2333	68.9465	25.6276
	(5.2010132, 0.427468599)	0.1753	87	4.2214	68.1266	25.5181
	(5.208881249, 0.427468599)	0.1755	87	4.2153	67.7170	25.4129
	(5.193145151, 0.418876401)	0.2665	133	5.3874	218.6349	32.8242
	(5.2010132, 0.418876401)	0.2673	133	5.3975	220.8565	32.7172
	(5.208881249, 0.418876401)	0.2667	133	5.3715	215.1896	32.6119
	(5.193145151, 0.4231725)	0.2667	133	5.3782	216.6434	32.9397
(0.0190, 0.05350)	(5.2010132, 0.4231725)	0.2673	133	5.3803	217.0896	32.8319
	(5.208881249, 0.4231725)	0.2674	133	5.3298	206.4131	32.7320
	(5.193145151, 0.427468599)	0.2669	133	5.3364	207.7665	33.0549
	(5.2010132, 0.427468599)	0.2679	133	5.3364	207.7662	32.9465
	(5.208881249, 0.427468599)	0.2663	133	5.2446	189.5425	32.8397
	(5.193145151, 0.418876401)	0.1705	85	4.5481	94.4502	12.8135
	(5.2010132, 0.418876401)	0.1717	85	4.5659	96.1526	12.6835
	(5.208881249, 0.418876401)	0.1714	85	4.5737	96.9082	12.5561
	(5.193145151, 0.4231725)	0.1707	85	4.5339	93.1204	12.9531
(0.03190, 0.09420)	(5.2010132, 0.4231725)	0.1716	85	4.5526	94.8831	12.8224
	(5.208881249, 0.4231725)	0.1705	85	4.5707	96.6184	12.6935
	(5.193145151, 0.427468599)	0.1710	85	4.5449	94.1528	13.0916
	(5.2010132, 0.427468599)	0.1716	85	4.5541	95.0209	12.9602
	(5.208881249, 0.427468599)	0.1714	85	4.5649	96.0495	12.8308

Appendix D

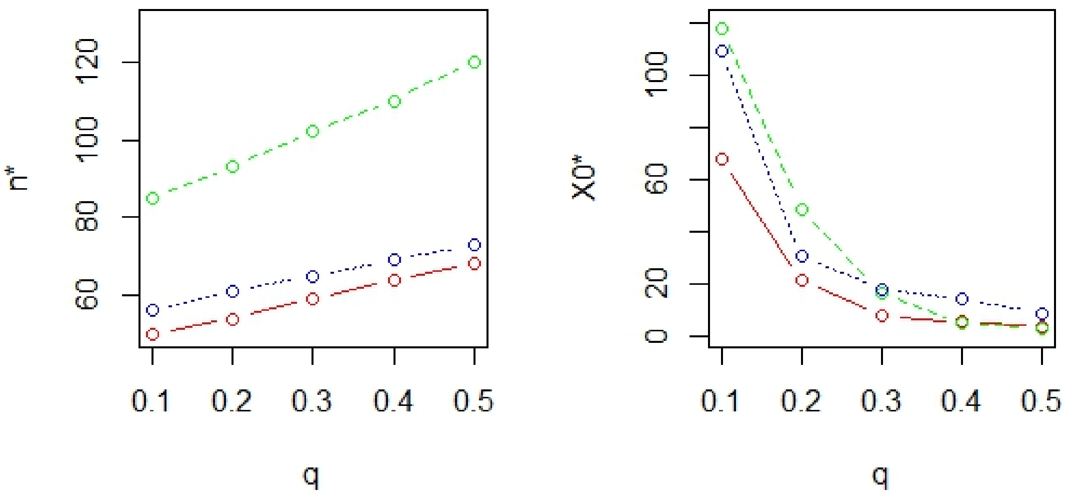

Figure 2: Change of optimal sample size and censoring time against the degree of censoring.

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